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Editorial: special issues on “frustrated spin systems and numerical methods” in commemoration of Johannes Richter

<https://doi.org/10.1515/zna-2026-0062>

Published online March 9, 2026

1 Johannes Richter

Johannes Richter, who passed away in May 2025, was a theoretical physicist who was internationally known for his contributions to our understanding of frustrated magnetism and for worldwide leading numerical procedures developed in his group. He grew up in Dresden where he also studied physics and did his PhD. In 1986, he submitted his habilitation thesis on “Theoretical investigations of electric and magnetic properties of disordered solids.” He spent a postdoc at Moscow State University and became a docent and later a professor of theoretical physics at Magdeburg University in the early 1990s. After his retirement, he enjoyed the hospitality of the Max Planck Institute for the Physics of Complex Systems (MPIPKS) in Dresden.

We, the two authors of this editorial, enjoyed the collaboration with Johannes Richter over more than 25 years. We are very thankful for the fruitful and pleasant interaction with him.

Johannes Richter started his career with amorphous ferromagnets [1]. He developed a taste for quantum spin systems and competing interactions (also known as “frustration”) early in his career, see [2]. Indeed, a large body of his later work was devoted to two-dimensional frustrated spin lattices such as the J_1 – J_2 square lattice [3] or the kagome lattice [4]–[6] to mention just two prominent examples. With his collaborators, he studied ground-state phase diagrams [7] as well as magnetization curves in order

to identify frustration effects [8], [9]. In the early 2000s, Johannes and his coauthors coined the term “independent localized magnons” [4], [10], which was later related to the flat bands in one-magnon space discovered by Tasaki and Mielke in the early 1990s [11], [12]. Today, flat-band physics is investigated in several areas of physics, see, e.g., [6], [13]–[22]. Flat-band systems are also created using cold atoms in optical lattices [23], [24] or by employing photonic lattices [25]–[27]. A review about the many facets of flat-band systems was written by Johannes Richter and coworkers in 2015 [21].

Johannes Richter was always eager to drive theoretical methods to the ultimate power possible with current computer technology. Among these efforts, we would like to highlight his contributions to high-temperature series expansions of magnetic observables [28]–[30] that were later combined with the entropy method suggested by Bernu and Misguish [31] and used in several studies [32]–[39].

Already in the early 2000s, Johannes Richter was interested in applications of the Green-function theory to the Heisenberg model [39]–[41]. Also in the early 2000s and motivated by discussions with Ray Bishop, Johannes Richter invested strongly in the coupled-cluster method [41]–[43]. The collaboration with Ray Bishop and Damian Farnell produced many results including a book they edited, which became a standard until today [44].

A fourth powerful method we want to mention is exact diagonalization by means of the Lanczos procedure, which was brought to perfection in the publicly available program *spinpack* [45] written by Jörg Schulenburg during his Ph.D. [46] with Johannes Richter. This program has been refined since then and used for many record calculations, see, e.g., [5], [47]. Later, it was extended toward the finite-temperature Lanczos method [48], [49], which allowed very accurate approximations of thermal expectation values in Hilbert spaces with dimensions of up to about 10^{11} [50].

Johannes Richter was always very interested in the work of others, in experimental results and applications [51]–[54], and in particular in the work of younger colleagues. A large fraction of the articles assembled in the present special issues thus stems from scientists that

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benefited from scientific discussions with Johannes Richter. He also fostered contacts to Eastern European colleagues among them in particular colleagues from L'viv/Ukraine. Several articles in the special issues are a living proof of these bonds.

2 Contributions to the special issue

In commemoration of Johannes Richter, the *Zeitschrift für Naturforschung A* publishes more than 20 original research articles in three issues. These articles provide a modern view on problems of frustrated magnetism as well as on current developments of methods used in the field. An introduction to the presented articles will be given in each issue separately.

The present issue of April 2026 contains the following articles.

1. The article of Florek, Schmidt, and Jaśniewicz-Pacer presents a comprehensive and rigorous analysis of the lowest energy configurations of classical spin polygons characterized by arbitrary couplings between neighboring spin sites. The authors show that these ground states exhibit either collinear or coplanar arrangements.
2. The present view of modern physics on nonequilibrium dynamics is that generic systems equilibrate or thermalize under rather general conditions, even closed systems under unitary time evolution. Schlüter, Schnack, and Ecksele address in their article the exception of nonergodic one-magnon magnetization dynamics of the kagome lattice antiferromagnet due to the existence of flat bands or equivalently independent localized magnons, which are eigenstates of the Hamiltonian and thus cannot equilibrate, i.e., spread across the lattice, see also [55].
3. Platonic or Archimedean bodies can constitute strongly frustrated spin systems, see [56], [57], which sometimes are realized as magnetic molecules [58], [59]. Their magnetic properties are often investigated by means of classical spin models [60], as in the contribution of Konstantinidis. He investigates the classical antiferromagnetic XX model for spins mounted on the vertices of the icosahedron and finds that the magnetization is characterized by two discontinuities as a function of the external field and that for a wide field range above the first discontinuity the ground state is degenerate.
4. Flat magnon bands can be generated or modified magnetoelectrically [61]. In the contribution by Ohanyan, a generalized sawtooth-chain model with three sets of exchange parameters (symmetric Heisen-

berg exchange, axial Ising anisotropy, and antisymmetric Dzyaloshinskii–Moriya (DM) exchange) is investigated where the effective DM interaction parameters are generated via the Katsura–Nagaosa–Balatsky (KNB) mechanism of magnetoelectricity. Several electric-field-induced flat-band scenarios in the distorted sawtooth chain are analyzed, and a mapping between the flat-band solutions for a general DM interaction and its specific KNB-induced form can be established.

5. Spin chains with competing interactions such as the delta chain can give rise to exciting phenomena such as quantum critical points [62], [63] where the ground-state degeneracy is macroscopic. In the example of the spin-1/2 J_1 – J_2 chain at $J_1/J_2 = -4$ [62], the quantum-critical point is accompanied by a proliferation of multi-magnon bound states in the presence of an external magnetic field, see, e.g., Ref. [64]. The contribution by Nishimoto revisits this problem and presents a detailed and systematic study of multi-magnon bound states in the spin-1/2 ferromagnetic–antiferromagnetic J_1 – J_2 chain in an external magnetic field using the density-matrix renormalization group method. This contribution reflects the interests of Johannes Richter in the thermodynamic properties close to the quantum critical point at $J_1/J_2 = -4$ [65], [66] and provides a comprehensive map of multi-magnon bound-state stability across field and frustration.
6. The contribution by Dmitriev, Krivnov, and Vasilyev investigates the spin-1/2 anisotropic Heisenberg model on diamond-decorated lattices where competing ferromagnetic and antiferromagnetic interactions again give rise to a macroscopic ground-state degeneracy. The authors first investigate the phase diagram and compute the ground-state entropy for the diamond chain. They then move on to a diamond-decorated square lattice, which they investigate numerically using a mapping to bond percolation. A similar mapping of the ground-state degeneracy in a percolation problem was previously applied to electron systems [67].
7. Karl'ová and Strečka investigate the Heisenberg model on the diamond-decorated honeycomb lattice in an external magnetic field. By combining density-matrix renormalization group calculations, sign-problem-free quantum Monte Carlo simulations in a dimer basis, Lanczos diagonalization, and an effective lattice-gas approach, they reveal a rich ground-state phase diagram with many magnetization plateaus and analyze the effect of thermal fluctuations. This work again stands in the tradition of collaborations with Johannes

Richter, specifically on diamond and octahedral chains [68], [69].

Acknowledgments: We would like to thank Jörg Schulenburg for restoring access to the legacy web pages [29], [43], [45].

Research ethics: Not applicable.

Informed consent: Not applicable.

Author contributions: Not applicable.

Use of Large Language Models, AI and Machine Learning Tools: None.

Conflict of interest: The authors state no conflict of interest.

Research funding: None.

Data availability: Not applicable.

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